Unit 2
Quadratic Functions

Day 1
Forms of Quadratic Functions

Examples. Examine the following functions. Which are quadratic functions? Why?

1 $f(x) = 3x + 7$

No

2. $g(x) = 4 - x^2$

Yes → input is squared

3. $h(x) = x^2 - \sqrt{x}$

No b/c highest power must be 2

4. $k(x) = 3x^2 - x^3$

No

& power must be a natural #

Recall that quadratic functions can be written in three different forms:

- **Vertex form:** $f(x) = a(x-h)^2 + k$
- **Factored form:** $f(x) = a(x-r_1)(x-r_2)$
- **Standard form:** $f(x) = ax^2 + bx + c$

Transforming equations into different forms
Algebra steps can be used to convert quadratic functions into different forms if careful attention is paid to the order of operations.

Examples. Identify the form of the following quadratic functions, then write the function in standard form.

5. $f(x) = -2(x+1)(x+4)$

Factored form

$f(x) = -2(x+1)(x+4)$
$f(x) = -2(x^2+4x+x+4)$
$f(x) = -2(x^2+5x+4)$
$\boxed{f(x) = -2x^2 - 10x - 8}$ Standard form

6. $g(x) = 2(x-6)^2 + 5$

Vertex form

$g(x) = 2(x-6)(x-6) + 5$
$g(x) = 2(x^2 - 6x - 6x + 36) + 5$
$g(x) = 2(x^2 - 12x + 36) + 5$
$g(x) = 2x^2 - 24x + 72 + 5$
$\boxed{g(x) = 2x^2 - 24 + 77}$ Standard

Examples: Write the following functions in factored form:

7. $m(x) = x^2 - 14x + 24$

$\boxed{m(x) = (x-12)(x-2)}$

24
$-2 \quad -12 = -14$

8. $h(t) = 2t^2 - 3t + 1$

$\boxed{h(t) = (2t-1)(t-1)}$

$2t^2 - 2t - t + 1$ ← check ✓

Assignment 2.1

Day 2

Finding Zeros of Quadratic Functions

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The zero (or root) of a function is a solution of the equation $f(x)=0$

Use the zero property to find zeros of quadratic functions from **factored form**: $a(x-r_1)(x-r_2)=0$

Examples. Find the zeros: (find x)

1. $f(x)=2(x-1)(x+5)$

$0=2(x-1)(x+5)$

$x-1=0 \quad x+5=0$

$+1 \quad +1 \quad -5 \quad -5$

$\boxed{x=1 \qquad x=-5}$ ← zeros

2. $f(x)=(x-11)^2$

$0=(x-11)(x-11)$ ← not neccessary to do twice.

$x-11=0$

$+11 \quad +11$

$\boxed{x=11}$

Finding zeros of quadratic functions from **vertex form:** $a(x-h)^2+k=0$

- Solve for $(x-h)^2$. (add over constant k, then divide by coefficient a.
- Take **square root** of both sides.
- Remember that there are TWO square roots for every real positive number.
- Solve for x and simplify (if possible).

Examples. Find the zeros:

3. $f(x)=3(x-2)^2-12$

set to zero → $0=3(x-2)^2-12$

$+12 \qquad +12$

$\frac{12}{3}=\frac{\cancel{3}(x-2)^2}{\cancel{3}}$

add in plus or minus ↘ $\pm\sqrt{4}=\sqrt{(x-2)^2}$

$\pm 2 = x-2$

$+2 \qquad +2$

$2\pm 2=x \Rightarrow \boxed{\begin{matrix}x=4\\x=0\end{matrix}}$

4. $f(x)=(x+1)^2-2$

$0=(x+1)^2-2$

$+2 \qquad +2$

$\sqrt{2}=\sqrt{(x+1)^2}$

$\pm\sqrt{2}=x+1$

$-1 \qquad -1$

$-1\pm\sqrt{2}=x$

Leave in root form.

$\boxed{x=-1\pm\sqrt{2}}$

Finding zeros of quadratic functions from **standard form**: $ax^2+bx+c=0$

- Factor if possible
- Otherwise, use the quadratic formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Simplify if possible. sing song to remember!

Examples. Find the zeros:

5. $f(x)=2x^2+3x-1$

$a=2 \quad b=3 \quad c=-1$

$x=\frac{-3\pm\sqrt{3^2-4(2)(-1)}}{2(2)}$

$x=\frac{-3\pm\sqrt{9--8}}{4}$

$\boxed{x=\frac{-3\pm\sqrt{17}}{4}}$ ← can't simplify further

6. $f(x)=x^2-4x+2$

$a=1 \quad b=-4 \quad c=2$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(2)}}{2(1)}$

$x=\frac{4\pm\sqrt{16-8}}{2}$

can simplify further → $x=\frac{4\pm\sqrt{8}}{2}$ → $x=\frac{\cancel{4}^{2}}{\cancel{2}}\pm\frac{\cancel{2}\sqrt{2}}{\cancel{2}}$

$\sqrt{8}$: 4, 2; (2 2)

$\boxed{x=2\pm\sqrt{2}}$

124-129

Quadratics and Complex Numbers

The **Fundamental Theorem of Algebra** was first proposed in the early 1600s but not proved until the 1800s. It states every polynomial function of degree *n* has exactly *n* zeros (root, solutions of $f(x)=0$). These solutions can be imaginary, complex $(a+bi)$ or real. Any zero may be multiple.

Quadratic functions are polynomial functions of degree 2, so every quadratic function has exactly two zeros. If there seems to be only one zero, it is really a **double zero.**

Examples. Find all zeros of the quadratic functions:

1. $k(x)=-2(x+3)^2$

$0=-2(x+3)(x+3)$

Set each term w/ any x equal to zero.

$x+3=0$
$-3 \quad -3$
$\boxed{x=-3}$

2. $v(x)=x^2-2x+5$

can't factor → quadratic formula.

$x=\dfrac{2\pm\sqrt{4-4(1)(5)}}{2(1)}$ $\boxed{x=1\pm 2i}$

$x=\dfrac{2\pm\sqrt{-16}}{2}=\dfrac{2}{2}\pm\dfrac{4i}{2}=1\pm 2i$

3. $f(x)=-3x(x-9)$

$0=-3x(x-9)$

$\dfrac{-3x}{-3}=\dfrac{0}{-3}$ $\boxed{x=0}$

$x-9=0$
$+9 \quad +9$
$\boxed{x=9}$

4. $f(x)=(x-5)^2+9$

$0=(x-5)^2+9$
$-9 \quad -9$

$\pm\sqrt{-9}=\sqrt{(x-5)^2}$

$\pm 3i=x-5$
$+5 \quad +5$

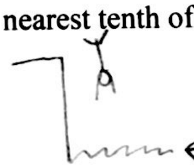
$5\pm 3i=x$ $\boxed{x=5\pm 3i}$

Using zeros to solve problems:

Examples.

5. A cliff diver dives off a cliff 80 feet above the water. The equation that gives his height *t* seconds after the dive begins is given by: $h(t)=-16t^2+80$. How many seconds will it take for the diver to hit the water (round answer to nearest tenth of a second)?

h(t) = height
t = seconds


← height = 0

$0=-16t^2+80$
$-80 \quad -80$
$\dfrac{-80}{-16}=\dfrac{-16t^2}{-16}$

$\sqrt{t^2}=\sqrt{5}$ ← no plus or minus neg time doesn't exist

$t=2.2$

$\boxed{2.2 \text{ seconds}}$

6. John holds a pistol straight upward and fires. The height of a bullet is a function of time and is described by: $h(t)=-16t^2+1200t$ where *t* is in seconds and *h* is in feet. How long does John have to move out of the way of the bullet falling from the sky?

h(t) = height
t = seconds


← height = 0

$0=-16t^2+1200t$

$0=-16t(t-75)$

$\dfrac{-16t}{-16}=\dfrac{0}{-16}$

$t-75=0$
$+75 \quad +75$

$t=0$ or $t=75$

zero seconds doesn't make sense

$\boxed{75 \text{ seconds to move.}}$

Assignment 2.3

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Day 4

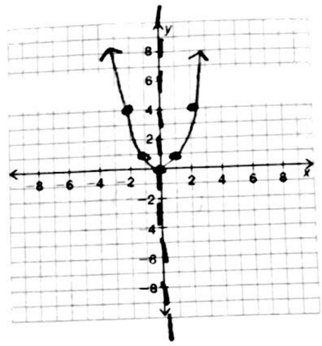
Graphing Quadratic Functions

Characteristics of quadratic graphs:

- Shape → parabola
- Vertex → rounded
- Symmetry → vertical
- Concave up/down smiling / frowning

Start with the Basics:

Examples. Consider the structure of $f(x)=x^2$ and its corresponding graph.
This is the parent function for the quadratic family of functions.



1. Complete the table. Then plot and label the points on the coordinate plane.

x	$f(x) = x^2$
0	0
1	1
2	4

$x=0$

2. Draw a dashed line to represent the axis of symmetry. Write the equation for the axis of symmetry. Then plot and label the symmetric points in quadrant II. Finally, draw a smooth curve to represent $f(x)=x^2$.

(-1, 1) (-2, 4)

Graphing from Vertex Form: $f(x)=a(x-h)^2+k$

- Vertex: (h, k)
- y-intercept is $f(0)$

Graphing from Factored Form: $f(x)=a(x-r_1)(x-r_2)$

- x-intercept(s): $(r_1, 0)$ and $(r_2, 0)$
- Vertex: x-coordinate of the vertex is half-way between the x-intercepts
- y-intercept is $f(0)$

Graphing from Standard Form: $f(x)=ax^2+bx+c$

- Vertex: $x=-\dfrac{b}{2a}$
- y- intercept is $(0, c)$

Graphing from All Forms:

- Use a to determine concavity (up/down) and dilation/compression
- Use symmetry to get additional points
- Use zeros to get x-intercepts

Examples. Graph. Identify (label) the vertex and two additional points:

– Standard form –

3. $f(x)=x^2+4x-2$

$a=1$ $b=4$ $c=-2$

Vertex:

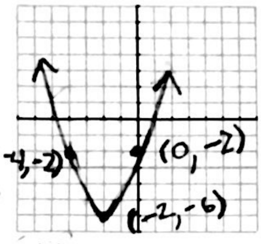
$x=\frac{-b}{2a}$

$x=\frac{-4}{2(1)}=-2$ $(-2,-6)$

Y = plug x into f(x)

$y=(-2)^2+4(-2)-2$

$y=4-8-2=-4-2=-6$



Y-int: (0,-2) use symmetry to find (-4,-2)

– vertex form –

4. $m(x)=-(x-4)^2+2$

Vertex: (4, 2)

y int: plug in zero for x.

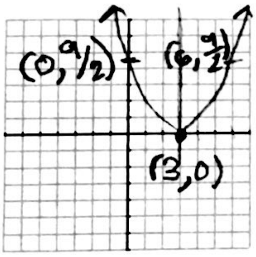
(0, ?) ⇒ (0,-14)

$-(0-4)^2+2$

$-(16)+2=-16+2=-14$

–vertex form–

5. $y=\frac{1}{2}(x-3)^2+0$



Vertex:

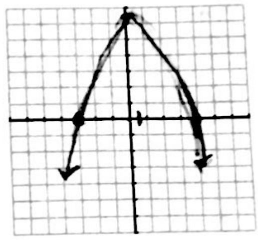
(3,0)

y-int:

$\frac{1}{2}(0-3)^2 \Rightarrow \frac{1}{2}(9)$

$(0, 9/2) \Rightarrow (0, 4.5)$

6. $h(x)=-2(x+3)(x-4)$ factored form



x-int:

(-3,0) (4,0)

vertex:

(1.5, 22.5)

$-2(1.5+3)(1.5-4)$

$-2(4.5)(-2.5)$

22.5

Using the vertex to solve problems

The vertex gives the maximum or minimum point of a quadratic function.

Examples.

7. Does the following function have a maximum or a minimum value? Identify the point that gives the maximum or minimum:

$$g(x)=-2(x-3)^2+4$$

Max: (3,4)

8. A chain store manager learns that the daily profit, P, is related to the number of clerks working that day, x, according to the equation $P(x)=-25x^2+300x$. What number of clerks will maximize the profit, and what is the maximum possible profit?

find max (vertex)

–Standard form–

$a=-25$ $b=300$

$x=\frac{-b}{2a}$

$\frac{-300}{2(-25)}=\frac{-300}{-50}=6$

$\boxed{\text{6 clerks}}$

Day 5

Transforming Functions

Transformations:

- **Rigid transformations (translations and reflections)**
- **Non-rigid transformations (dilations)**

Given a function $f(x)$:

$f(x+c)$ left c units

$f(x-c)$ right c units

$f(x)+c$ up c units

$f(x)-c$ down c units

$f(-x)$ reflect over y axis

$-f(x)$ reflect over x axis (flip down, frowning face)

$cf(x)$ Vertical Stretch by c if c>1, vertical shrink by c if 0<c<1 (fraction)

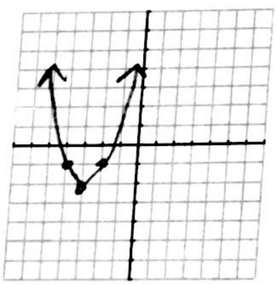
$f(cx)$ Horizontal Stretch by c if 0<c<1, horizontal shrink by c if c>1

Transformations can be used to graph quadratic functions. Start with the basic points for the parent function: (0, 0), (1, 1) and (2, 4). Identify the transformations and then shift, reflect, and/or adjust the points and draw the graph.

When there are multiple transformations, do reflections/dilations first, then shifts.

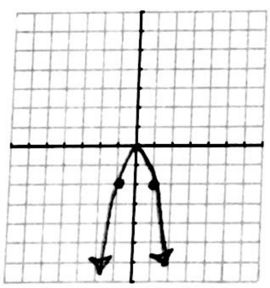
Examples. Given $f(x)=x^2$, state the transformations and graph the new function.

1. $g(x)=f(x+3)-2$



left 3, dow 2

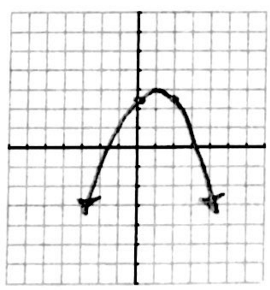
2. $h(x)=-2f(x)$



reflect over x-axis

V-Stretch by 2

3. $j(x)=-0.5f(x-1)+3$



reflect over x-axis

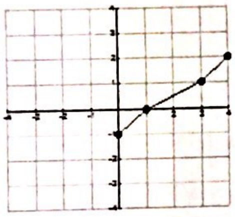
V. shrink by 1/2

right 1, up 3

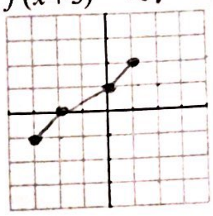
Transformations can be used to graph other types of functions, as well as quadratics.

Examples. Determine the transformations and use the graph of f to sketch each new function.

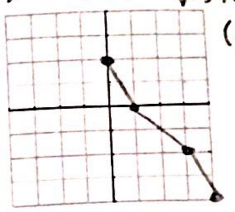
$y = f(x)$



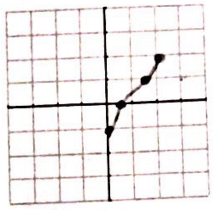
4. $f(x+3)$ left 3



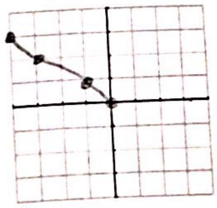
5. $y = -2f(x)$ flip over x V. stretch by 2 (multiply y by 2)



6. $y = f(2x)$ H. shrink by 2 (divide x by 2)



7. $y = f(-x)+1$ flip over y up 1



Writing functions "in terms of"

$k(x) = 3(x+1)^2 - 4$ is written in terms of x

$g(x) = f(x-4)$ is written in terms of $f(x)$

$m(x) = -r(x)+1$ is written in terms of $r(x)$

Example 8. If $f(x) = x^2$, write an equation for $g(x)$ ***in terms of*** $f(x)$ that will produce the following transformations:

a. Shifted left 4

$g(x) = f(x+4)$

b. Shifted up 6

$g(x) = f(x) + 9$

c. Reflected across the x-axis

$g(x) = -f(x)$

d. Vertical stretch by a factor of 2

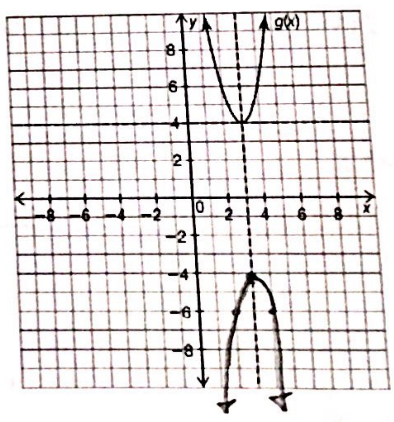
$g(x) = 2f(x)$

e. Horizontal compression by a factor of 3 (shrink)

$g(x) = f(3x)$

Examples. Given the graph of $g(x) = 2(x-3)^2 + 4$

9. Graph $m(x) = -g(x)$.



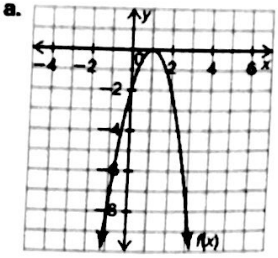
10. Rewrite $m(x)$ in terms of x

$m(x) = -2(x-3)^2+4$

Examples.

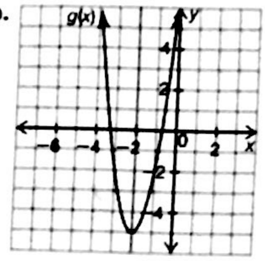
11. If $h(x) = x^2$, write an equation in terms of x for the transformed graphs.

a.



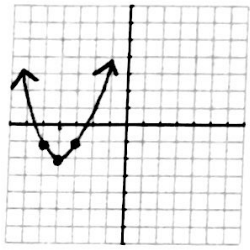
$f(x) =$ $-2(x-1)^2$

b.



$g(x) =$ $3(x+2)^2 - 5$

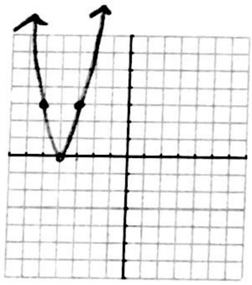
12. Given $f(x) = x^2$, use transformations to sketch the graph of $d(x) = f(x+4) - 2$.



Rewrite $d(x)$ in terms of x.

$d(x) = (x+4)^2 - 2$

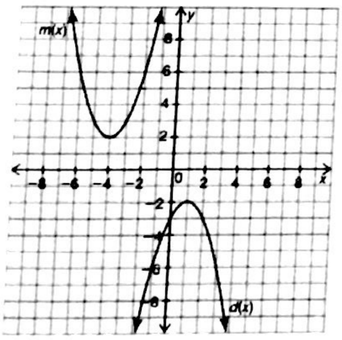
13. Given $f(x) = x^2$ Use transformations to sketch the graph of $d(x) = 3(x+4)^2$



Rewrite $d(x)$ in terms of $f(x)$.

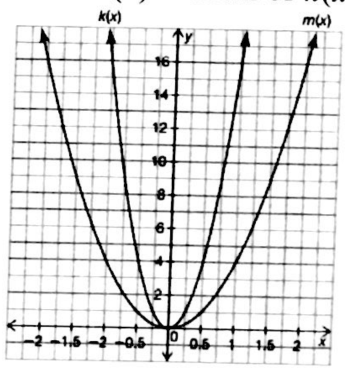
$d(x) = 3f(x+4)$

14. Write $m(x)$ in terms of $d(x)$.



$m(x) = -d(x+5) + 4$

15. Write $m(x)$ in terms of $k(x)$.



$m(x) = k(\frac{1}{2}x)$

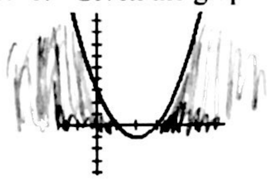
Assignment 2.5

Day 6

Quadratic Inequalities

Quadratic inequalities can be solved by finding the factored form of an equivalent quadratic function and graphing from the zeros and the a value. **The vertex is not important.** The solutions are the x-values that give y-values that make the inequality true.

Example. 1. Given the graph of $f(x)$ shown below, solve $f(x) \geq 0$. Write solution in inequality notation.



$x \leq 1$ or $x \geq 3$

Examples. Solve the following quadratic inequalities. Show a graph to justify answers.

2. $x^2 > 9$

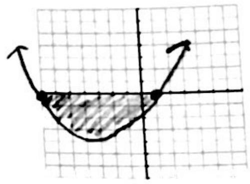
$x^2 - 9 > 0$

$(x+3)(x-3) > 0$

$x > 3$ or $x < -3$

3. $x^2 + 6x - 7 \leq 0$

$(x-1)(x+7) \leq 0$

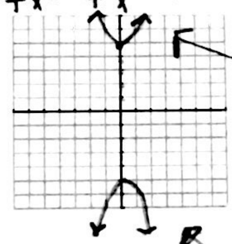


$-7 \leq x \leq 1$

4. $-x^2 > 25$

$+x^2 \quad +x^2$

$0 > x^2 + 25$



$x^2 + 25 < 0$

$\boxed{\text{No Solution}}$

$-x^2 - 25 > 0$

or if you subtracted 25

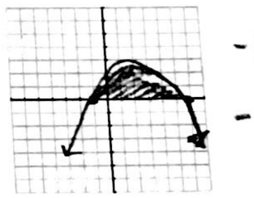
Still no solution!

5. $-x^2 + 5x > -6$

$-x^2 + 5x + 6 > 0$

$-(x^2 - 5x - 6) > 0$

$-(x-6)(x+1) > 0$



$\boxed{-1 < x < 6}$

Assignment 2.6

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Day 7

Unit 2 Review

Assignment 2.7

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Day 8

Unit 2 Test ***All late/absent assignments due for Unit 2***

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Complex Number Operations

Review: Write 3^5 in two different ways as a product of two numbers each raised to a power. Describe the power rule used.

$3^2 \cdot 3^3 = 3^5$ add exponents $3^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 = 3^5$

When you solve $x^2 = 25$, you get two answers:

$\sqrt{x^2} = \sqrt{25} \rightarrow x = \pm\sqrt{25} \rightarrow \boxed{x = \pm 5}$

What about $x^2 = -25$?

$\sqrt{x^2} = \sqrt{25}$ ← can't take sq. rt. of neg. number.

Powers of *i*

$i = \sqrt{-1}$

$i^2 = \left(\sqrt{-1}\right)^2 = -1$

So, what is i^4 ?

$i^4 = i^2 \cdot i^2 = (-1)^2 \cdot (-1)^2 = 1 \cdot 1 = 1$

Memorize: $i = i$ $i^3 = -i$
$i^2 = -1$ $i^4 = 1$

Knowing i, i^2, i^4, and the power rule for exponents, you can simplify i raised to different powers.

$18 = 4 \cdot 4 + 2$ $41 = 4 \cdot 10 + 1$

Examples. Simplify the following powers of i:

1. i^3

$i^3 = i^2 \cdot i = (-1) \cdot i = -i$

2. i^{18}

$i^{18} = (i^4)^4 \cdot i^2 = (1)^4 \cdot (-1) = \boxed{-1}$

3. i^{41}

$i^{41} = (i^4)^{10} \cdot i^1 = (1)^{10} \cdot i = \boxed{i}$

Square Roots of Negative Numbers : $i = \sqrt{-1}$

Examples. Simplify each square root:

4. $\sqrt{-36}$

$6i$

5. $\sqrt{-8}$

$2i\sqrt{2}$

6. $5 + \sqrt{-13}$

$5 + i\sqrt{13}$

Imaginary numbers:

numbers w/ just i. $2i$, $i\sqrt{3}$, i, $81i$, $2i\sqrt{3}$, ...

Complex numbers:

numbers w/ a real part & imaginary part $6 + i$, $2 + i\sqrt{3}$, ...

Working with Complex Numbers

Complex numbers can be added, subtracted or multiplied, similarly to polynomials.

Treat i sort of like a variable when adding / multiplying / subtracting.

Examples. Perform the operation. Write answer in $a + bi$ form (standard form). Show your work.

7. $(3+2i)-(1-6i)$

$3 + 2i - 1 + 6i$

$\boxed{2 + 8i}$

8. $4i + 3 - 6 + i - 1$

$4i - 3 + i - 1$

$\boxed{-4 + 5i}$

9. $5i(9-2i)$

$45i - 10i^2$

$45i - 10(-1)$

$\boxed{10 + 45i}$

10. $(3+5i)(2-3i)$

$6 - 9i + 10i + 15\cancel{i^2}$

$\boxed{21 + i}$

Assignment 2.2